08.05.2023

De Mazenod Collage - Kadana

G.C.E. (Advance Level) Examination

Term Test - 2023 **Grade - 12**

Combined Maths - I **Time - $2\frac{1}{2}$ hrs.**

Part A

Answer all the questions.

1) Using the principle of mathematical induction, Prove that $\sum_{r=1}^{n}(3r-2) = \frac{n(3n-1)}{2}$ for all $n \in \mathbb{Z}^+$.

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2) Using the Principle of Mathematical Induction, Prove that $n^3 + 5n$ is divisible by 3 for every $n \in \mathbb{Z}^+$.

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3) The remainder when the polynomial $x^4 + ax^2 + 3x + b$ is divided by $x^2 + x$ is -5. Find the value of a and b.

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4) Show that $\lim_{x \to \frac{\pi}{6}} \frac{\tan\left(x - \frac{\pi}{6}\right)}{\sqrt{6x} - \sqrt{\pi}} = \frac{\sqrt{\pi}}{3}$.

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5) Let $f(x) = kx^2 - 2k^2x + k$, where $k \neq 0$, find the values of k. Such that $f(x) > 0$ for all the real values of x.

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6) Let $\sin \propto = \frac{1}{\sqrt{10}}$ and $\cos\beta = -\frac{1}{\sqrt{5}}$ for $\frac{\pi}{2} < \propto < \pi$ and $\frac{\pi}{2} < \beta < \pi$ respectively. Find the value of $\sin(\propto + \beta)$.

7) Show that the equation of the tangent line to the reangular hyperbola parametrically given by $x = ct$ and $y = \frac{c}{t}$ for $t \neq 0$, at the points $P \equiv \left(cp . \frac{c}{p}\right)$ is given by $x + p^2y = 2cp$. The normal line to this hyperbola at P meets the hyperbola again at another point $Q \equiv \left(cq . \frac{c}{q}\right)$. Show that $p^3q = -1$.

8) Let, $A + B = \frac{\pi}{6}$. Show that $(\sqrt{3} + \tan A)(\sqrt{3} + \tan B) = 4$. Hence, deduce that, $\tan\frac{\pi}{12} = 2 - \sqrt{3}$.

9) Show that $(1+x^2)\frac{d^2y}{dx^2}+(2x-1)\frac{dy}{dx}=0$ when $y=e^{tan^{-1}(x)}$

10) Show that the equation of tangent line is drawn on the curve $\frac{x^2}{25}-\frac{y^2}{9}=1$ at the point $P\equiv(5\sec\theta,3\tan\theta)$ where $0<\theta<\frac{\pi}{2}$ is $(3\sec\theta)\,x-(5\tan\theta)y=15$. Find the $y\ axis-$ intercept of tangent line which is drawn at point $(5\sqrt{2},3)$ on the above curve.

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Part B

11)

a) The equation $x^2 + mx + 15 = 0$ has roots α and β and the equation $x^2 + hx + k = 0$ has roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$.

i. Write down the value of k.

ii. Find an expression for h in terms of m.

iii. Find the two possible values of α such that $\beta - 2\alpha = 1$

iv. Hence, find the two possible values of m and h.

b) Let $f(x) = ax^4 + bx^3 + x^2 + cx - 14$, where $a, b, c \in \mathbb{R}$. If $(x-1)$ and $(x-2)$ are factors of $f(x)$ and when $f(x)$ is divided by $(x+1)$, the remainder is 6. Find the values of a, b, c. Write $f(x)$ as a product of linear factors and find the remainder when $f(x)$ is divided by $(3x-1)$.

12)

a) If $f(r) = \{r(r+1)\}^2$, prove that $f(r0 - f(r-1) = 4r^3$

Hence, deduce that $\sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$.

Write down the r^{th} term Ur of the series $\frac{3}{1^3} + \frac{5}{1^3+2^3} + \frac{7}{1^3+2^3+3^3} + \cdots$

Find $v(r)$ such that $U(r) = v(r) - v(r+1)$.

Show that $\sum_{r=1}^{n} U_r = 4 - \frac{4}{(n+1)^2}$

Show that the infinite series $\sum_{r=1}^{n} U_r$ is convergent.

b) Given $A = \begin{pmatrix} 5 & 2 \\ -7 & -3 \end{pmatrix}$

i. Find A^2 and find B such that $B = 3I + A - A^2$

ii. Calculate AB

iii. Deduce A^{-1}

13)

a) Let $f(x) = \frac{x(x-3)}{(x-2)^2}$, for $x \neq 2$.

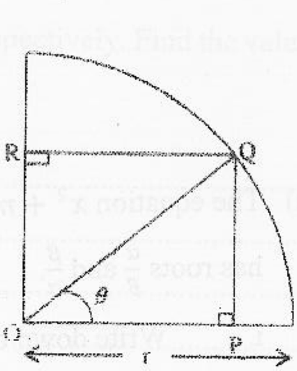
Show that $f'(x)$ the derivative of $f(x)$, is given by

$f'(x) = \frac{6-x}{(x-2)^3}$ for $x \neq 2$

Hence, find the increasing interval and decreasing interval of $f(x)$. Find the coordinates of turning points.

It is given that $f''(x) = \frac{2(x-8)}{(x-2)^4}$ for $x \neq 3$.

Find the coordinates of the inflection point. Sketch the graph of $y = f(x)$ indicating the asymptotes turning points and inflection points.



b) A rectangle OPQR is drawn into the quarter circle with radius 'r' as shown in the figure. O is the centre. Given that $P\hat{O}Q = \theta$. Show that area A of the rectangle $OPQR$ is given by $A = \frac{r^2}{4}\sin 2\theta$. Show that A is maximum at $\theta = \frac{\pi}{4}$.

14)

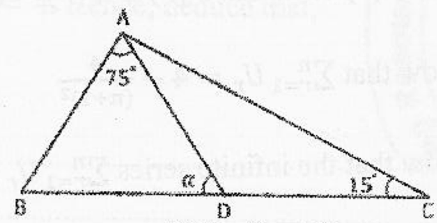
a) Write the expansion of $\sin(A+B), \sin(A-B)$

Prove that

i) $\sin(90^0 - \theta) = \cos\theta$

ii) $\sin 2\theta = 2\sin\theta\cos\theta$

iii) Show that $\sin 75^0 = \frac{\sqrt{6}+\sqrt{2}}{4}$ and $\sin 15^0 = \frac{\sqrt{6}-\sqrt{2}}{4}$ hence deduce that $\sin 75^0 \sin 15^0 = \frac{1}{4}$



b) In a given triangle ABC, D is the midpoint of BC as shown in the figure.

$A\hat{C}D = 15^0$ and $B\hat{A}D = 75^0$. Let $A\hat{D}B = \alpha^0$

By using the sine rule for the appropriate triangles.

Show that $\sin(\alpha - 15^0)\sin(105^0 - \alpha) = \sin 75^0 \sin 15^0$

Explain $\sin(105^0 - \alpha) = \cos(\alpha - 15^0)$

By using the results obtained above in part (a) Deduce that $\alpha = 30^0$